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14. ABSTRACT
A novel oceanic test has revealed an interesting dynamical limit for the hydrostatic Navier-Stokes equation that argues against using high-order methods for simulating these types of flows. This strange behavior was uncovered largely due to the fidelity of the spectral element model to the mathematical setting of the hydrostatic equations. A non-hydrostatic version of the spectral element model has been produced that incorporates many of the advances developed in recent years; the model is currently being applied to simulating gravity currents at high Reynolds number. We have continued in our efforts to build an unstructured-grid ocean modeling community by organizing several national and international meetings where participants share their common experience in developing and running unstructured grid models.

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High-Order, Multi-Scale Ocean Modeling on Adaptive, Unstructured Meshes: Comparison of SEOM and ROMS in the Northwest Atlantic

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1 Goals

Our goals are the development and validation of an unstructured ocean modeling capabilities. This includes implementing and improving various numerical algorithms; the development and dissemination of validation tests ranging from idealized process oriented ones to realistic ones ; and the building of a user and development community for unstructured grid model development.

2 Results

2.1 Validation Tests: The Winant Problem

Our recent model validation focussed primarily on a process-oriented study of the wind-driven circulation in a well-mixed estuary in the presence of topography and rotation, Winant [2004]. The reference solution was provided by an asymptotic analysis of the linearized and hydrostatic Navier-Stokes equations. A notable feature of this asymptotic solution is that, when the horizontal and vertical viscous coefficients are of the same order, a reasonable assumption for non-stratified flows, the *horizontal* component of the viscous forces drops out on account of $D \ll L$ where D and L are the vertical and horizontal scales of motion. The same argument, $D/L \ll 1$ is of course behind the hydrostatic approximation. The asymptotic solution did not concern itself with the lateral boundaries of the domain, and concentrated on matching the solutions in the interior of the basin. The ROMS simulation reproduced quickly the results expected from the asymptotic simulation whereas SEOM's results showed some strange behavior. Further examination of the problem revealed that the boundary conditions of the problem were overspecified. In fact the only permissible conditions on the lateral sides is the specification of the depth integrated transport; the vertical velocity profile was solely determined by the top and surface boundary conditions. Thus there was no mechanism in the simplified equations to impose the vertical profile of the velocity entering/leaving the domain, and simple no-throgh flow boundary conditions could not be imposed. It turns out that SEOM was in fact reacting reasonably to the mathematical

Figure 1: *Density contours in a dam-break type simulation of a gravity current in a closed channel using a non-hydrostatic spectral element model. The discretization consisted of 128×32 spectral elements of polynomial degree 14 for the vorticity and streamfunction, and 13 for the density. The Froude number for this simulation is 1/2, the Reynolds number is 10,000 and the Pradtl number is 7.*

setting of the problem thanks to its high-order numerical schemes, and helped uncover some unexpected behavior of the hydrostatic Navier-Stokes equations.

The problem could be seen a classical case of a singular perturbation problem where the neglect of the highest order derivative, the horizontal viscous term, permitted the imposition of only a subset of the boundary conditions. An additional physical mechanism must be invoked to impose the no-normal flow condition. The analysis shows that including either horizontal viscous forces *or* non-hydrostatic effects remedies the situation. In the former case however, it is the horizontal viscous term that slows-down the normal flow, whereas in the non-hydrostatic case, this is achieved, as expected, via a pressure gradient *without* involving frictional forces. In both cases the recirculation boundary layer introduced scaled like the geometris aspect ratio, D/L, and hence remain subgrid scale with respect to the resolution used in most basin scale models.

The aforementioned results have important implications for *all* lateral basin boundaries, including those located on the offshore side of coastal ocean models. Velocity boundary conditions must be applied at these open boundaries to allow flow exchange with the rest of the basin; these are usually obtained either from observation or from a nested run. Our analysis shows that *horizontal* mixing plays an important role in permitting the exchange of information, and that numerically-induced narrow recirculation zones may be introduced.

The role of stratification in the balance of forces needs to be assessed for the results to hold more generally. Two important factors are then introduced, namely the baroclinic pressure gradient which could in fact provide a compensating pressure gradient, and the suppression of vertical mixing relative to the horizontal one. The latter enhances the role of horizontal mixing so that its magnitude is comparable to the vertical one. These issues complicate the analysis, and future work may have to proceed with numerical experimentation.

A byproduct of this validation problem has been the development of a non-hydrostatic model that is currently being applied to simulating gravity currents at very high Reynolds number. Most of the algorithms in this model have been in development for years, most notably

the efficient elliptic solver, and the Discontinuous Galerkin discretization of the the tracer equations. Figure 1 illustrates the sharpness of the features captured in a dam-break type simulation, most notably the vigorous mixing taking place between the light and dense fluids. These simulations are supporting the validation of mixing parametrization, and are designed to provide reference solution for very high Reynolds number flows. Recently we have begun to investigate the impact of unsteady forcing on the mixing efficiency. This work is being performed in collaboration with Dr. Tamay Ozgokmen and a student. This code is currently being enhanced in several ways to handle larger domain with an aspect ratio of 1:50 instead of simply 1 to 5 so that the evolution of the gravity current over longer distance, of order 50 km, can be pursued.

3 Community Building

We have continued in our efforts to develop and nurture an international commuity of model users and model developers. These efforts revolved primarily around a series of workshops, The International Workshop on Unstructured Mesh Numerical Modelling of Coastal, Shelf and Ocean flows, that have been organized annually since 2002, and where participants share their common experience and their developmental efforts. A series of special issues of the *Ocean Modelling* journal appeared that summarized the contributions made during the meetings. The 2006 meeting took place in Miami and was organized by Dr. Iskandarani with collaboration from Drs. Julia Levin and Joannes Westerink. The participants in these workshops span a large spectrum of model users and developers, namely those interested in estuarine, coastal and basin scale models. Research groups representing various approaches to unstructured grid discretization (finite volume and finite element) are present at these meetings and contribute actively to the discussions. The 2007 meeting will take place in London and we are planning to continue our participation. In addition Dr. Julia Levin organized a separate meeting in Washington DC for US-based research groups to discuss the state of the art in unstructured grid ocean modeling.

4 Publications

The following publications summarize our work to date:

- J. Pietrzak, M. Iskandarani, J. Schröter and F. Lyard, "Special Issue: The third international workshop on unstructured mesh numerical modelling of coastal, shelf and ocean flows Toulouse, France, September 20–September 22, 2004", *Ocean Modelling*, **15**, Issues 1–2, pp 1-2, 2006.

- J.C. Levin, M. Iskandarani, and D.B. Haidvogel, "To continue or discontinue: comparisons of continuous and Discontinuous Galerkin formulations in a Spectral Element Ocean Model", *Ocean Modelling*, **15**, Issues 1–2, 56–70, 2006.

- J.C. Levin, D.B. Haidvogel, B. Chua, A.F. Bennett and M. Iskandarani, "Euler-Lagrange equations for the spectral element shallow water system", *Ocean Modelling*, **12**, Issues 3-4, pages 348-377, 2006.
- M. Iskandarani, J. Levin, B.-J. Choi and D.B. Haidvogel, "Comparison of advection schemes for high-order $h - p$ finite element and finite volume methods", *Ocean Modelling*, **10**, 232–252 (2005).
- M. Iskandarani, D. B. Haidvogel, J. C. Levin, and E.P. Chassignet, *The importance of being viscous; or, how the hydrostatic primitive equations enforce lateral boundary conditions*, to be submitted.

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Clinton D. Winant. Three-dimensional wind-driven flow in an elongated, rotating basin. *Journal of Physical Oceanography*, 34(2):462–476, 2004.